LETTER

Error Bound of Collision Probability Estimation in Non-saturated IEEE 802.11 WLANs*

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SUMMARY We analytically prove that the error in the channel idle time-based collision probability estimation in face of non-saturated stations is bounded by $2/(CW_{min}+1)$ in the IEEE 802.11 wireless LANs (WLANs). This work explicitly quantifies the impact of non-saturation, and the result vindicates the use of the estimation technique in real-life IEEE 802.11 WLANs, in such applications as the acknowledgement-based link adaptation and the throughput optimization through contention window size adaptation.

key words: *IEEE 802.11, collision probability, non-saturated model, contention window, clear channel assessment*

1. Introduction

In the IEEE 802.11 wireless LAN (WLAN) [1], it is useful to precisely determine the probability of collision P_c in a transmission failure event. For instance, we can use the estimated value for the acknowledgement (ACK)-based link adaptation algorithms [2], [3] or the optimal contention window size adaptation [4]. In case of the operation under an impaired channel, however, it is difficult to directly observe P_c since transmission failures can be also caused by the channel. A practical way to overcome this problem is to obtain P_c by way of the channel idle time t_i through the assistance of the Clear Channel Assessment (CCA) in the IEEE 802.11 physical (PHY) layer, a core functionality for the CSMA/CA access. For instance, [4] shows how to obtain P_c analytically from t_i. However, analysis works on the IEEE 802.11 WLAN including [4] almost always assume *saturated* stations, which means the stations are infinitely backlogged. Although it facilitates the analysis, it is hardly a realistic assumption. Therefore, there is a need to validate them against the more realistic non-saturation assumption.

In this letter, we quantify the impact of non-saturation in the P_c estimation via t_i. Specifically, we prove that the estimation of P_c from t_i has a small error bound of $2/(1 + CW_{min})$ for uniformly *non-saturated* station population. This result vindicates the use of the technique of using t_i to estimate P_c for such optimization schemes as the acknowledgement-based link adaptation and the optimal contention window size determination.

2. Derivation of P_c from t_i

The probability that a slot is idle when there are n saturated stations is given by $P_i = (1-\tau)^n$, where τ is the probability that a given station attempts transmission in the slot. Notice that

$$P_c = 1 - (1-\tau)^{n-1} \Rightarrow \tau = 1 - (1-P_c)^{\frac{1}{n-1}}. \tag{1}$$

The average idle time, t_i, which is the number of idle slots, becomes

$$t_i = \frac{1}{1-P_i} - 1 = \frac{1}{1-(1-P_c)^{\frac{n}{n-1}}} - 1. \tag{2}$$

As to n, we know from [5] that

$$n = 1 + \frac{\log(1-P_c)}{\log\left(1 - \frac{2(1-2P_c)}{(1-2P_c)(W+1)+P_c W(1-(2P_c)^m)}\right)}, \tag{3}$$

where W denotes the minimum contention window size CW_{min} and m denotes the maximum backoff stage parameters of the IEEE 802.11 medium access control (MAC) layer standard. Combining (1) and (2) to eliminate n, we can numerically obtain P_c as a function of t_i. Figure 1 plots P_c versus t_i, and we show that the numerical derivation (solid line) closely matches the simulation performed with the 802.11b MAC implementation in ns-2 (dotted line) [6]. The number of stations corresponding to the idle time ranges from 1 (far right) to 40 stations (far left) in the figure.

Now, we deal with the P_c estimation in the non-saturated stations environment. Given the idle time measurement of t_i, let the estimation error due to the non-saturated stations be

$$\epsilon(t_i) = P_c(t_i) - P_c^u(t_i), \tag{4}$$

where P_c^u and P_c are the collision probability with non-saturated stations and saturated stations, respectively. In Fig. 1, the marks are from non-saturated scenarios, where each station is uniformly loaded by 0.2 Mbps traffic to 0.5 Mbps of constant bit rate (CBR) traffic carried over User Datagram Protocol (UDP). Under different bit rates, the number of stations that generates the given idle time differs. The larger the bit rate, the smaller the number of stations that yields the given idle time. For instance, the plotted range for 0.2 Mbps in the figure is 1 to 34 stations, while for 0.5 Mbps,

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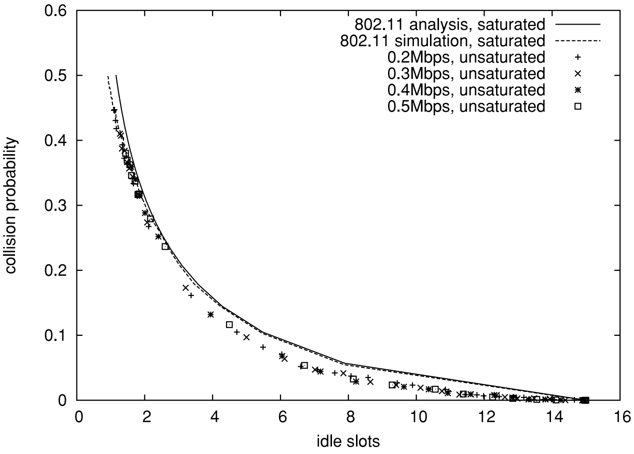

Fig. 1 P_c versus t_i.

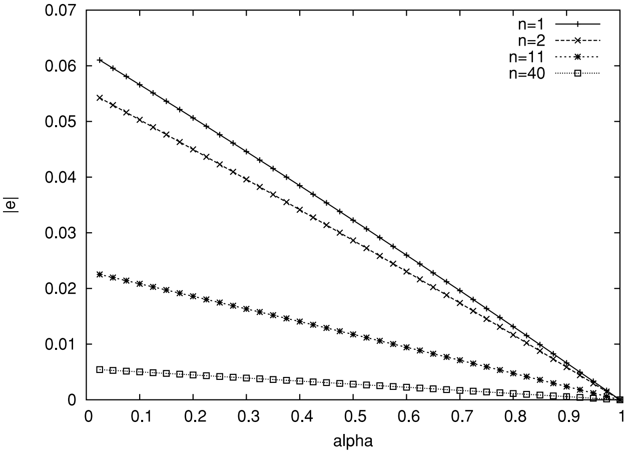

Fig. 2 $|\epsilon| = |P_c - P_c^u|$ versus α.

it ranges 1 to 18 stations. The simulation shows that for the given t_i, the difference in P_c from the saturated scenario is indeed small for all t_i values.

Now we analytically prove that ϵ in (3) is bounded. Let τ $[\tau_u]$ and n $[n_u]$ be per-station transmission probability in a given slot and the station population under the saturated [non-saturated] assumption, respectively. And let $\tau_u = \alpha\tau$, with $\alpha \leq 1$. The smaller the α, the less saturated the stations are. In particular, $\alpha = 1$ denotes the saturated stations system. Under the same t_i, and from (1) and (2), we have

$$\frac{1}{1-(1-\tau_u)^{n_u}} - 1 = \frac{1}{1-(1-\tau)^n} - 1.$$
$$\Rightarrow (1-\tau_u)^{n_u} = (1-\tau)^n. \tag{5}$$

Thus, the difference between the collision probabilities is

$$\epsilon = (1-\tau_u)^{n_u-1} - (1-\tau)^{n-1} = X\left[\frac{(\alpha-1)\tau}{(1-\alpha\tau)(1-\tau)}\right], \tag{6}$$

where

$$X = (1-\tau_u)^{n_u} = (1-\tau)^n,$$

and n is the number of stations in the saturated system that *would* lead to the same t_i as n_u generates with the given α. Notice ϵ is a non-positive value since $\alpha \leq 1$. Thus $|\epsilon|$ decreases with α, and converges to 0 for $\alpha \to 1$. Also, τ is a slow decreasing function of n. Thus in (6), $X = (1-\tau)^n$ as well as the absolute value of the fractional term decreases with n. Therefore, the error bound occurs when $\alpha = 0$ and $n = 1$ with a given t_i (where $\alpha = 0$ corresponds to the inactivity of the terminal). But then, in (6) we have

$$\epsilon = -\tau.$$

We know from [7] that

$$\tau = \frac{2}{1 + W + P_c W \sum_{i=0}^{m-1} (2P_c)^i}. \tag{7}$$

But since $n = 1$ implies $P_c = 0$, (7) becomes

$$\tau = \frac{2}{1+W}.$$

Therefore, $|\epsilon|$ is bounded by $\frac{2}{1+W}$. In the IEEE 802.11b MAC, for instance, $|\epsilon|$ is bounded by $1/16 = 0.0625$ since $W = CW_{min} = 31$.

Finally, in order to see the impact of n and α, we numerically compute P_c from n in (3), and with these two we derive τ in (1). Then, with n and τ in (6) we get ϵ. Figure 2 plots $|\epsilon|$ as a function of α for various n values for the IEEE 802.11b. It clearly shows that the magnitude of the estimation error decreases with α and/or n. In particular, for $\alpha = 0$ and $n = 1$, it is indeed bounded by 0.0625, and has smaller values for larger α or n values, which is usually the operating condition for real-life IEEE 802.11 WLANs.

3. Conclusion

In this letter, we quantify the error in the collision probability estimation due to non-saturation stations to be small and bounded. The property vindicates the use of idle time-based collision probability estimation in real-life IEEE 802.11 WLANs, for optimization techniques such as the acknowledgement-based link adaptation and the optimal contention window size determination.

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